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$$(\pm1)^{n-1}x=(\pm1)^{n-1}(y\pm1)=\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\pm1)^{n-1}y+(\pm1)^n$$

$$(\pm1)^n=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\pm1)^n.$$

By adding, and simplifying the coefficients of y, we have

$$K=y^n\pm(n+1)y^{n-1}+\tfrac{1}{2}[(n+1)n]y^{n-2}\pm\ \ldots\ldots+(\pm1)^{n-1}\tfrac{1}{2}[(n+1)n]y+(\pm1)^n(n+1),$$

which has binomial coefficients of the $(n+1)$th order. Substituting A, B, C, ... for the coefficients and restoring the values of y,

$$K=(x\mp1)^n\pm A(x\mp1)^{n-1}+B(x\mp1)^{n-2}\pm\ldots\ldots\ldots\ldots+(\pm1)^{n-1}B(x\mp1)+(\pm1)^nA.$$

[Expanding and combining the terms of the second member, we get the first member for a result. ZERR.]

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

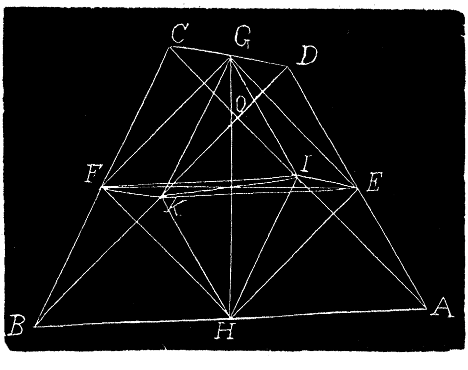
SOLUTIONS OF PROBLEMS.

43. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

The consecutive sides of a quadrilateral are a, b, c, d. Supposing its diagonals to be equal, find them and also the area of the quadrilateral.

II. Solution by A. H. BELL, Hillsboro, Illinois.

The solution as published simply demonstrates this theorem, $a^2+b^2+c^2+d^2=2x^2+4\overline{IK}^2$, with two unknowns, and is then solved for a particular case.



Let the sides of the quadrilateral AB, BC, CD, and AD, be a, b, c, and d; and the diagonals each $=2x$; $x+y$, $x-y$=the segments AO and OC; and BO and $OD=x+z$ and $x-z$. In the triangles AOB, BOC, and COD we have $(x+y)\cos A+(x+z)\cos B=a$, $\cos A=(4x^2+a^2-b^2)/(4ax)$, $\cos B=(4x^2+a^2-d^2)/(4ax)$, making

$z(4x^2+a^2-d^2)=(2a^2+b^2+d^2)x-8x^3-(4x^2+a^2-b^2)y$ ....(1).

Similarly BOC, $z(4x^2+b^2-c^2)=(2b^2+a^2+c^2)x-8x^3-(a^2-b^2-4x^2)y$ .....(2),

and COD, $z(-4x^2+b^2-c^2)=(b^2+2c^2+d^2)x-8x^3+(c^2-d^2+4x^2)y$ ....(3).

Subtracting (2) from (1), $(a^2-b^2+c^2-d^2)z=(a^2-b^2-c^2+d^2)x-8x^2y$ .....(4).

Subtracting (3) from (2), $8x^2z=(a^2+b^2-c^2-d^2)x-(a^2-b^2+c^2-d^2)y$.....(5).

Equating the values of z in (4) and (5) and solving for y, after letting

$(a^2-b^2+c^2-d^2)=e$, $a^2-b^2-c^2+d^2=f$, $a^2+b^2-c^2-d^2=g$, we have

$$y=(eg-8fx^2)/(e^2-64x^4).x \text{......................(6).}$$

Equating the values of z and solving for y in (1) and (4) after letting

$$a^2-b^2=m,\quad a^2-d^2=n,\quad 2a^2+b^2+d^2=p,\quad 2e+f=q,$$

and noting that $2n-e=g$, we have

$$y=(4qx^2-ep+fn)/(32x^4+4gx^2-em).x \text{...............(7).}$$

Equating the values of y in (6) and (7),

$$512x^6-64(a^2+b^2+c^2+d^2)x^4+16[a^2(b^2-2c^2+d^2)+b^2(c^2-2d^2)+c^2d^2]x^2$$
$$+4(ac-bd)(ac+bd)(a^2-b^2+c^2-d^2)=0 \text{.......(8).}$$

Let $x^2=\frac{1}{8}y$, $4x^2=\frac{1}{2}y$, $2x=\sqrt{\frac{1}{2}y}$, or multiply (8) by the geometrical series, $\frac{1}{512}$, $\frac{1}{64}$, $\frac{1}{8}$, and 1, ratio=8, then (8) becomes

$$y^3-(a^2+b^2+c^2+d^2)y^2+2[a^2(b^2-2c^2+d^2)+b^2(c^2-2d^2)+c^2d^2]y$$
$$+4(ac-bd)(ac+bd)(a^2-b^2+c^2-d^2)=0 \text{........(9).}$$

When this is solved by Cardan's formula, then since there are given the sides of the triangles ABC and ACD, we have in the general formula, with the sides a, b, and $2x$, and c, d, and $2x$, the area for the quadrilateral

$$ABCD=\tfrac{1}{4}\sqrt{[(a+b)^2-4x^2]\times[4x^2-(a-b)^2]}$$
$$+\tfrac{1}{4}\sqrt{[(c+d)^2-4x^2]\times[4x^2-(c-d)^2]}.$$

A better transformation for (8) is to put $4x^2=y$, $2x=\sqrt{y}$. We then get $y^3-\frac{1}{2}(a^2+b^2+c^2+d^2)y^2$ $+\frac{1}{2}[a^2(b^2-2c^2+d^2)+b^2(c^2-2d^2)+c^2d^2]y+\frac{1}{2}(ac+bd)(ac-bd)(a^2-b^2+c^2-d^2)=0$.

Example: a, b, c, d=6, 5, 3, 4, respectively, in (9) gives

$$y^3-86y^2+894y-1216=0.$$

By Horner's method, $y=75.7270176+$ Diagonal $2x=6.1533+$ agreeing with a close drawing.

[Mr. Bell sent us this solution March 14, 1895. We have looked it over carefully and believe that it is entirely correct. The solution published in the July-August number of Vol. II is of a particular case. EDITOR.]

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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

57. Proposed by F. M. McGAW, A. M., Professor of Mathematics in Bordentown Military Institute, Bordentown, New Jersey.

Solve the following equation : $(1+x^2)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0.$

I. Solution by WILLIAM E. HEAL, A. M., Member of the London Mathematical Society, and Treasurer of Grant County, Marion, Indiana.

Let $y=bx\left[\int zdx+c\right]$ and the equation becomes

$$x(1+x^2)\frac{dz}{dx}+2z=0,\ \text{ or } \frac{dz}{z}+\frac{2dx}{x(1+x^2)}=0.$$

$\therefore\ z=c'(1+[1/x^2])$; $y=bx\{c'(x-[1/x])+c\}$, $=Bx+A(1-x^2)$.

II. Solution by H. C. WHITAKER, M. E., Ph. D., Professor of Mathematics in Manual Training School, Philadelphia, Pennsylvania.

Proceeding to obtain a solution in series, both values of y are found to terminate immediately. The complete primitive is $y=Ax+B(x^2-1)$.

III. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

It is shown (*Forsyth's Differential Equations*, Article 58) that

$$d^2y/dx^2+P(dy/dx)+Qy=R\ \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

gives, when $y=vw$ ....(2).